

3.13-3.16 Review

Key

Name _____ Date _____ Period _____

Find the exact value of each expression. Leave answers in terms of π when necessary.

1. $\sin^{-1}(-0.5)$

$$-\frac{\pi}{6}$$

2. $\arctan(-1)$

$$-\frac{\pi}{4}$$

3. $\sec^{-1}(\sqrt{2})$

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4$$

4. $\cos\left(\arcsin\left(\frac{1}{2}\right)\right)$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

5. $\tan\left(\arccos\left(\frac{\sqrt{2}}{2}\right)\right)$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

6. $\sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

$$-\frac{\pi}{4}$$

7. $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

8. $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

9. $\csc^{-1}\left(\sec\left(\frac{\pi}{3}\right)\right)$

$$\csc^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Find the exact value of each expression in degrees.

10. $\sin^{-1}(1) = 90^\circ$

11. $\arccos\left(-\frac{1}{\sqrt{2}}\right) = 135^\circ$

12. $\cot^{-1}(\sqrt{3}) = 30^\circ$

13. $\operatorname{arccot}(0) = 90^\circ$

Find all real numbers in $[0, 2\pi]$ that satisfy each equation.

14. $\cos(x) + 1 = 0$

$$\cos x = -1$$

$$x = \pi$$

15. $2\sin(x) - 1 = 0$

$$\sin x = \frac{1}{2}$$

$$x = \pi/6, 5\pi/6$$

16. $2\tan(x) + 2 = 0$

$$\tan x = -1$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Find all angles in $[0^\circ, 360^\circ]$ that satisfy each equation.

17. $2\sin(x) + 1 = 0$

$$\sin x = -\frac{1}{2}$$

$$x = 210^\circ, 330^\circ$$

18. $2\cos(x) = \sqrt{2}$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = 45^\circ, 315^\circ$$

19. $\sqrt{3}\tan(x) - 1 = 0$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$x = 30^\circ, 210^\circ$$

Find all angles in $[0^\circ, 360^\circ]$ that satisfy each equation. Round approximations to the nearest tenth of a degree.

20. $\sin \alpha = -0.244$

$$\alpha = \sin^{-1}(-0.244)$$

$$\alpha = -14.1^\circ + 360^\circ = 345.9^\circ$$

$$+ \alpha = 194.1^\circ$$

21. $\cos \alpha = -0.158$

$$\alpha = \cos^{-1}(-0.158)$$

$$\alpha = 99.1^\circ$$

$$\alpha = 260.9^\circ$$

22. $\cot \alpha = -0.433$

$$\alpha = \tan^{-1}(-0.433)$$

$$\alpha = -46.6^\circ + 180^\circ = 113.4^\circ$$

$$\alpha = 293.4^\circ$$

Find all real numbers in $[0, 2\pi]$ that satisfy each equation.

23. $2\sin(2x) - \sqrt{3} = 0$

$$\sin 2x = \frac{\sqrt{3}}{2}$$

$$\frac{2x}{2} = \frac{\pi}{3} + 2\frac{\pi k}{2}, \frac{2x}{2} = \frac{2\pi}{3} + 2\frac{\pi k}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, x = \frac{\pi}{3}, \frac{4\pi}{3}$$

24. $2\cos(2x) + \sqrt{3} = 0$

$$\cos 2x = -\frac{\sqrt{3}}{2}$$

$$\frac{2x}{2} = \frac{5\pi}{6} + 2\frac{\pi k}{2}, \frac{2x}{2} = \frac{7\pi}{6} + 2\frac{\pi k}{2}$$

$$\frac{2x}{2} = \frac{2\pi}{6} + 2\frac{\pi k}{2}, \frac{2x}{2} = \frac{4\pi}{6} + 2\frac{\pi k}{2}$$

$$x = \frac{5\pi}{12}, \frac{7\pi}{12}, x = \frac{2\pi}{12}, \frac{4\pi}{12}$$

25. $\sqrt{2}\cos\left(\frac{x}{2}\right) - 1 = 0$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$2\left(\frac{x}{2} = \frac{\pi}{4} + 2\pi k\right), 2\left(\frac{x}{2} = \frac{7\pi}{4} + 2\pi k\right)$$

$$x = \frac{\pi}{2}$$

$$x = \frac{7\pi}{2}$$

26. $\sqrt{3}\tan\left(\frac{x}{2}\right) + 1 = 0$

$$\tan\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{3}}$$

$$2\left(\frac{x}{2} = \frac{5\pi}{6} + \pi k\right)$$

$$x = \frac{5\pi}{3}$$

Find all real numbers in $[0, 2\pi]$ that satisfy each equation. Round approximate answers to the nearest tenth.

27. $2\tan^2 x = \tan x$

$$2\tan^2 x - \tan x = 0$$

$$\tan x(2\tan x - 1) = 0$$

$$\tan x = 0$$

$$x = 0, \pi$$

$$\tan x = \frac{1}{2}$$

$$x = \tan^{-1}\left(\frac{1}{2}\right)$$

$$x \approx .46 \approx .5$$

$$x \approx 3.6$$

28. $2\sin^2 x + \sin x = 1$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2}, \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$29. 6\sin^2 x + 5\cos x = 7$$

$$6\sin^2 x + 5\cos x - 7 = 0$$

$$6(1 - \cos^2 x) + 5\cos x - 7 = 0$$

$$6 - 6\cos^2 x + 5\cos x - 7 = 0$$

$$-1(-6\cos^2 x + 5\cos x - 1 = 0)$$

$$6\cos^2 x - 5\cos x + 1 = 0$$

$$(2\cos x - 1)(3\cos x - 1) = 0$$

$$\cos x = 1/2 \quad \cos x = 1/3$$

$$x = \pi/3, 5\pi/3 \quad x = 1.2, 5.1$$

$$31. \cos x \cos 2x - \sin 2x \sin x = \frac{1}{2}$$

$$\cos(x+2x) = \frac{1}{2}$$

$$\cos(3x) = \frac{1}{2}$$

$$\frac{3x}{3} = \frac{\pi/3 + 2\pi k}{3}$$

$$\frac{3x}{3} = \frac{5\pi/3 + 2\pi k}{3}$$

$$x = \frac{\pi}{9} + \frac{2\pi}{3}$$

$$x = \frac{5\pi}{9} + \frac{2\pi}{3}$$

$$x = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9} \text{ and } x = \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$$

$$30. \sin\left(\frac{\pi}{6}\right)\cos x - \cos\left(\frac{\pi}{6}\right)\sin x = -\frac{1}{2}$$

$$\sin\left(\frac{\pi}{6} - x\right) = -\frac{1}{2}$$

$$\frac{\pi}{6} - x = -\frac{1}{2}$$

$$\frac{\pi}{6} - x = \frac{7\pi}{6} \text{ and } \frac{\pi}{6} - x = \frac{11\pi}{6}$$

$$-x = \pi$$

$$x = -\pi + 2\pi = \pi$$

$$-x = \frac{10\pi}{6}$$

$$x = -\frac{5\pi}{3} + 2\pi$$

$$32. \sin(2\theta) = 3\sin(\theta)$$

$$2\sin\theta\cos\theta - 3\sin\theta = 0$$

$$\sin\theta(2\cos\theta - 3) = 0$$

$$\sin\theta = 0$$

$$\theta = 0, \pi$$

$$\cos\theta = \frac{3}{2} > 1$$

So no solution

$$34. 2\cos^2\theta + 1 = 3\cos\theta$$

$$2\cos^2\theta - 3\cos\theta + 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta - 1) = 0$$

$$\cos\theta = 1/2 \quad \cos\theta = 1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}, 0$$

$$33. 9\sin^2\theta + 6\sin\theta + 1 = 0$$

$$(3\sin\theta + 1)^2 = 0$$

$$\sin\theta = -\frac{1}{3}$$

$$\theta = \sin^{-1}(-1/3) \approx -.3398 + 2\pi$$

$$\theta = \pi + .3398 = 3.5$$

35. A block is set in motion hanging from a spring and oscillates about its resting position at $x = 0$ according to the friction $x = 0.6\sin 2t + 0.4\cos 2t$, where x is in centimeters and t is in seconds. For what values of t in the interval $[0, 3]$ is the block at its resting position $x = 0$?

$$0 = 0.6\sin 2t + 0.4\cos 2t$$

$$-0.6\sin 2t - 0.4\cos 2t$$

$$\frac{-0.6\sin 2t}{\cos 2t} = \frac{0.4\cos 2t}{\cos 2t}$$

$$\frac{-0.6\tan 2t}{-0.6} = \frac{0.4}{-0.6}$$

$$\tan 2t = (-\frac{2}{3})$$

$$2t = \tan^{-1}(-2/3)$$

$$2t = -.588 + \pi k$$

$$\frac{2t}{2} = \frac{-.588 + \pi k}{2}$$

$$t = -.29 + \frac{\pi}{2}$$

$$t = 1.28 \text{ and } 2.85, \text{ seconds}$$